Heat and Mass Transfer: Fundamentals & Applications
Fourth Edition
Yunus A. Cengel, Afshin J. Ghajar
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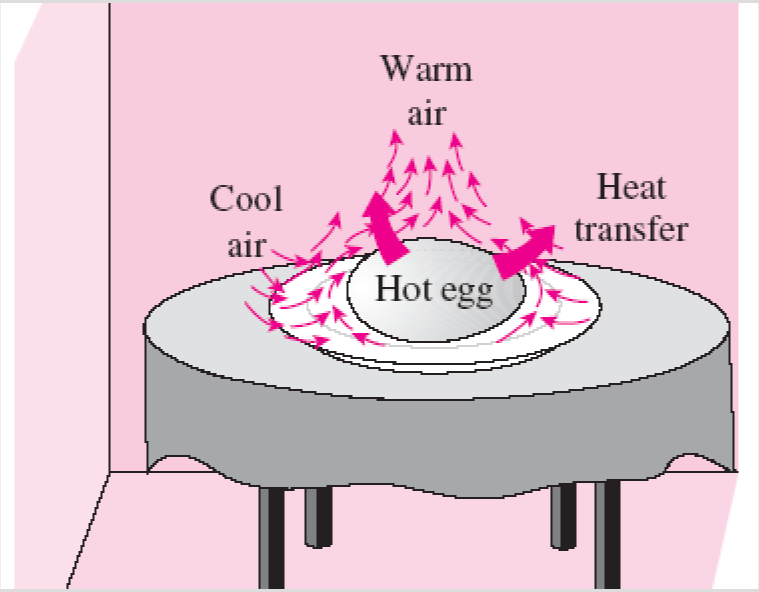
Chapter 9
NATURAL CONVECTION

PHYSICAL MECHANISM OF NATURAL CONVECTION

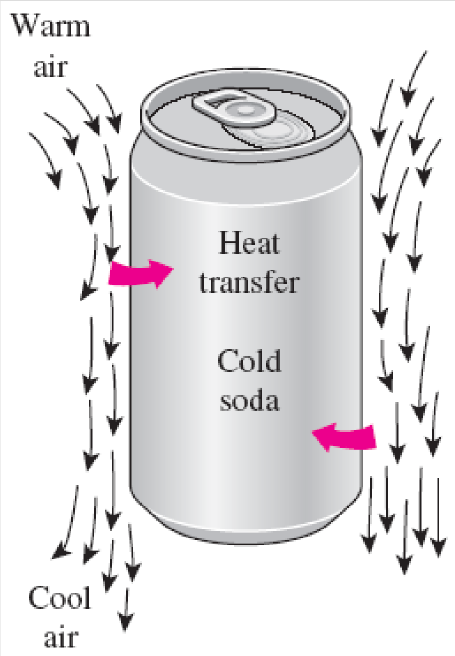
Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Examples?

Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current, and the heat transfer that is enhanced as a result of this current is called natural convection heat transfer.



The cooling of a boiled egg in a cooler environment by natural convection.



The warming up of a cold drink in a warmer environment by natural convection.

Buoyancy force: The upward force exerted by a fluid on a body completely or partially immersed in it in a gravitational field. The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body.

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}}$$

The net vertical force acting on a body

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, g V_{\text{body}} - \rho_{\text{fluid}}\, g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g V_{\text{body}} \end{aligned}$$

Archimedes' principle: A body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces.

The "chimney effect" that induces the upward flow of hot combustion gases through a chimney is due to the buoyancy effect.

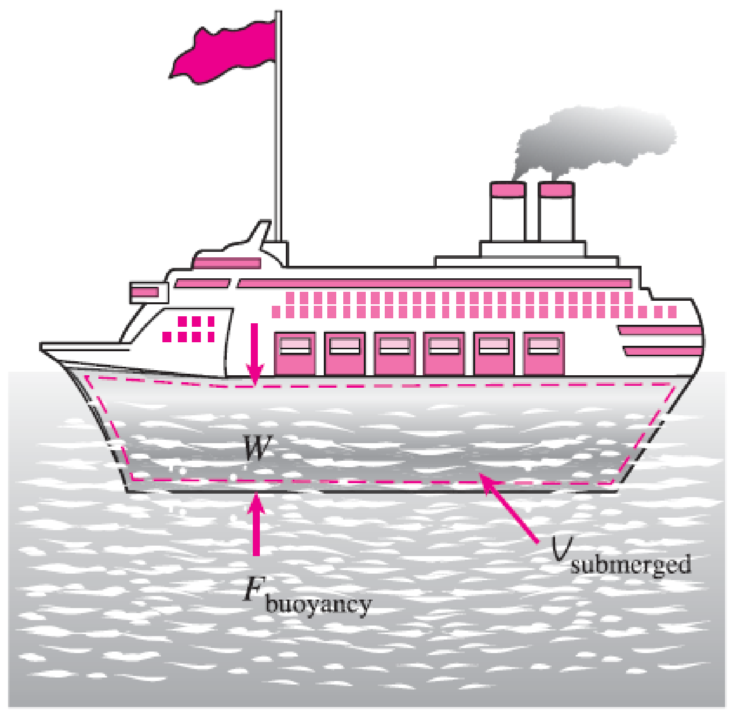
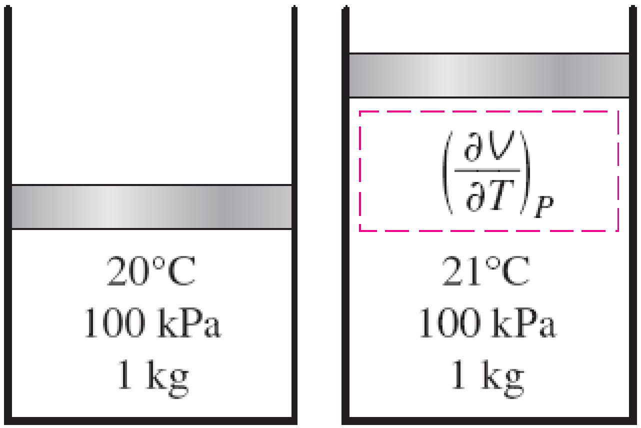
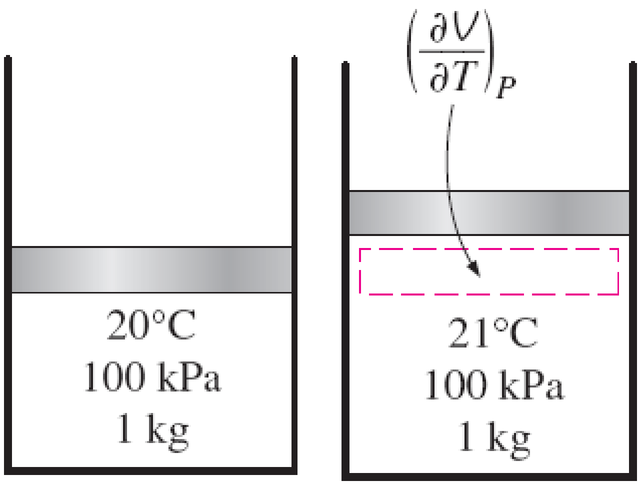


FIGURE 9–3
It is the buoyancy force that keeps the ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).



(*a*) A substance with a large β



(*b*) A substance with a small β

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

Volume expansion coefficient: Variation of the density of a fluid with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\text{K})$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad (\text{at constant } P)$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad (\text{at constant } P)$$

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K})$$

ideal gas

$(P = \rho RT)$

The larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

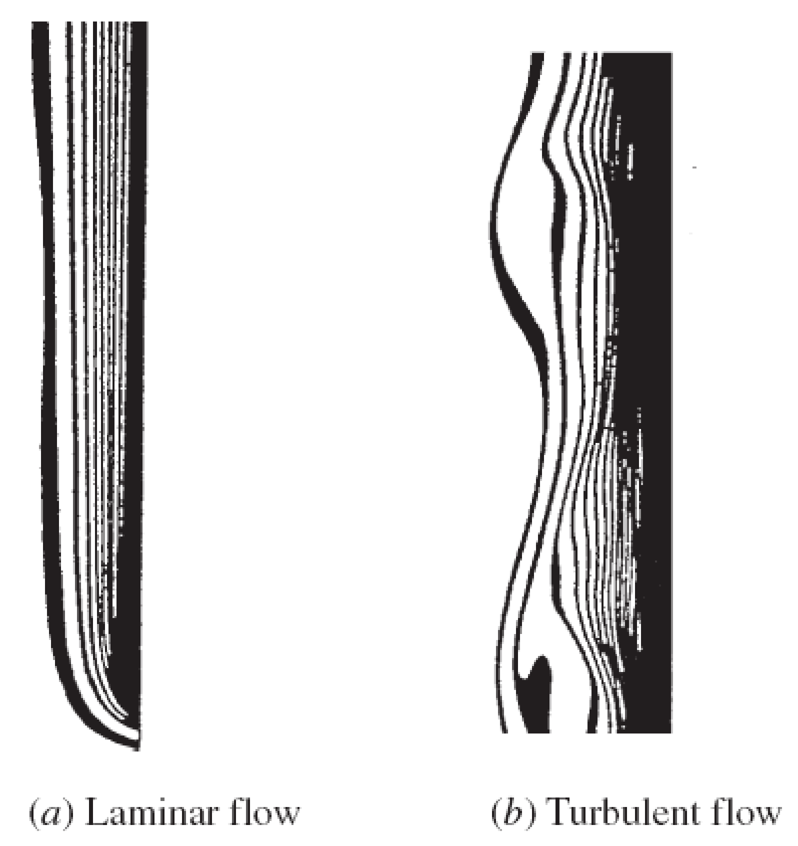
In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally.

The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction*.

An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature.*

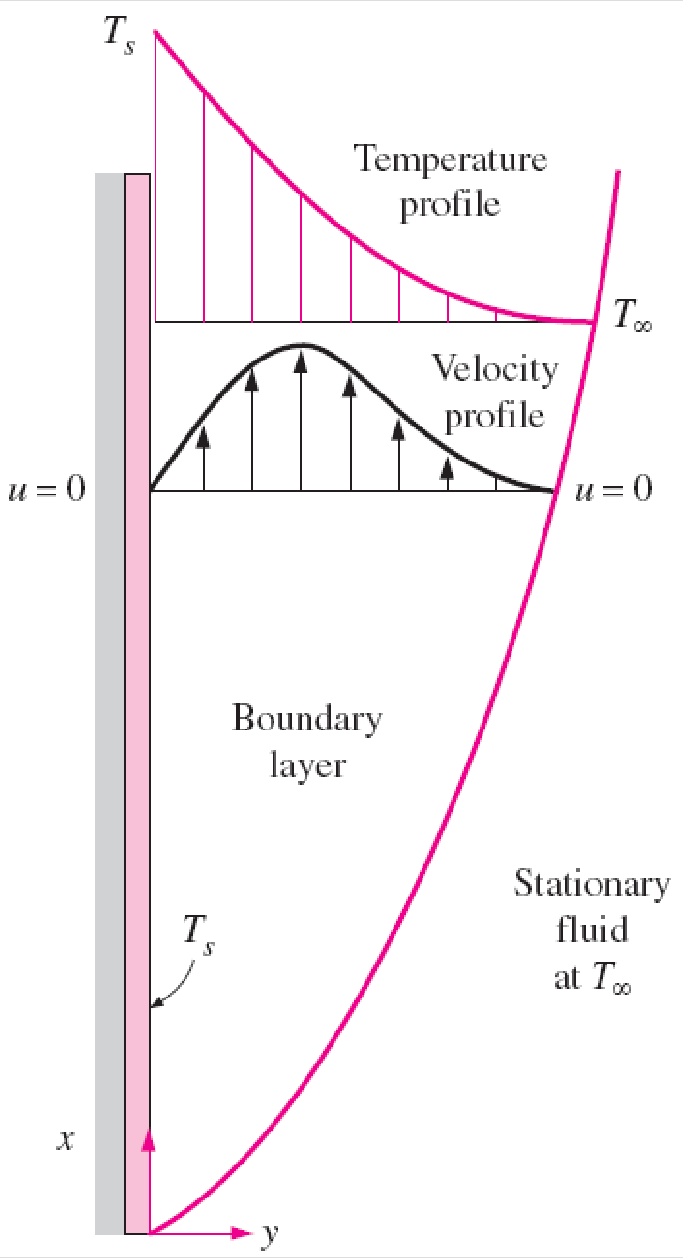
The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.*

The lines are closest near the surface, indicating a *higher temperature gradient.*



Isotherms in natural convection over a hot plate in air.

EQUATION OF MOTION AND THE GRASHOF NUMBER



The thickness of the boundary layer increases in the flow direction.

Unlike forced convection, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate.

At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface.

In the case of *cold surfaces,* the shape of the velocity and temperature profiles remains the same but their direction is reversed.

Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

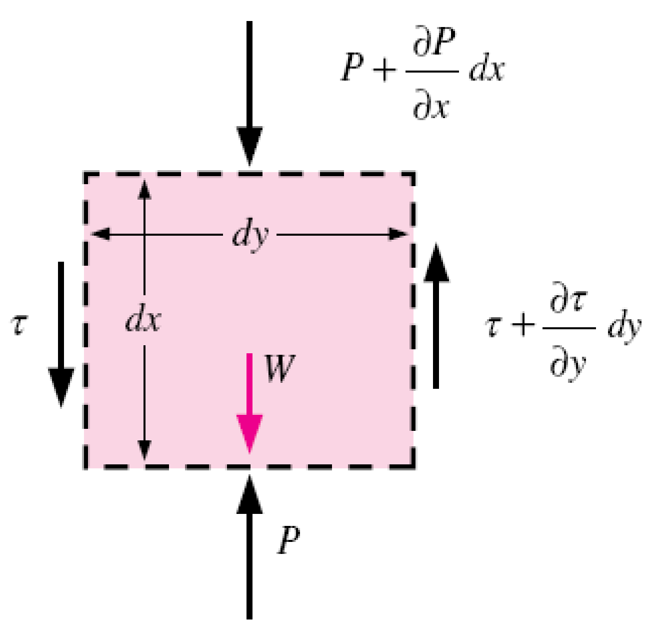
Derivation of the equation of motion that governs the natural convection flow in laminar boundary layer

$$\delta m \cdot a_x = F_x$$

$$\delta m = \rho(dx \cdot dy \cdot 1)$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$F_x = \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1)$$
$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1)$$



Forces acting on a differential volume element in the natural convection boundary layer over a vertical flat plate.

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. The momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The complete set of conservation equations, continuity (Eq. 6–39), momentum (Eq. 9–13), and energy (Eq. 6–41) that govern natural convection flow over vertical isothermal plates are:

Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

with the following boundary conditions (see Fig. 9–6):

At $y = 0$: $u(x, 0) = 0,\ v(x, 0) = 0,\quad T(x, 0) = T_s$

At $y \rightarrow \infty$: $u(x, \infty) \rightarrow 0,\ v(x, \infty) \rightarrow 0,\quad T(x, \infty) \rightarrow T_\infty$

The above set of three partial differential equations can be reduced to a set of two ordinary nonlinear differential equations by the introduction of a similarity variable. But the resulting equations must still be solved along with their transforred boundary conditions numerically.

The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities:

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

Substituting them into the momentum equation and simplifying give

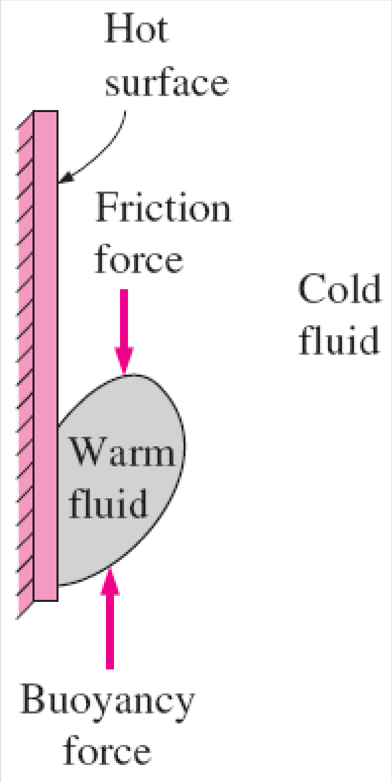
$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right] \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

Grashof number: Represents the natural convection effects in momentum equation

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

- The Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.
- For vertical plates, the critical Grashof number is observed to be about 10^9.

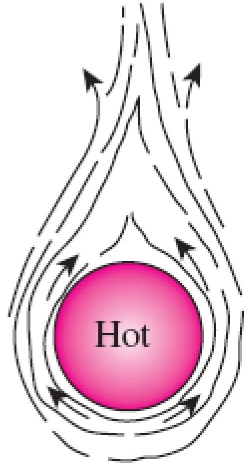


The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

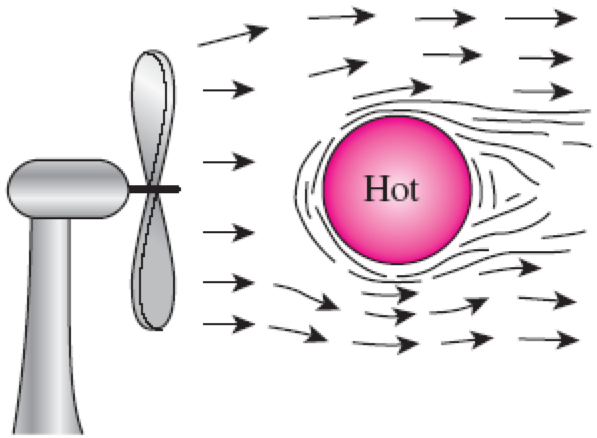
When a surface is subjected to external flow, the problem involves both natural and forced convection.

The relative importance of each mode of heat transfer is determined by the value of the coefficient Gr/Re^2:

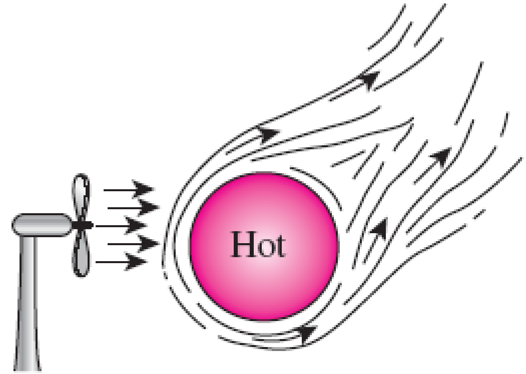
- Natural convection effects are negligible if $Gr/Re^2 \ll 1$.
- Free convection dominates and the forced convection effects are negligible if $Gr/Re^2 \gg 1$.
- Both effects are significant and must be considered if $Gr/Re2 \approx 1$ (mixed convection).



(*a*) Natural convection ($Gr_L / Re_L^2 >> 1$)



(*b*) Forced convection ($Gr_L / Re_L^2 << 1$)



(*c*) Mixed convection ($Gr_L / Re_L^2 \approx 1$)

FIGURE 9–10
The relative importance of convection heat transfer regimes for flow near a hot sphere.

NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation, the variation of temperature on the surface and the thermophysical properties of the fluid involved.

With the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies.

$$\mathrm{Nu} = \frac{hL_c}{k} = C(\mathrm{Gr}_L\,\mathrm{Pr})^n = C\,\mathrm{Ra}_L^n$$

$$\mathrm{Ra}_L = \mathrm{Gr}_L\,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$$

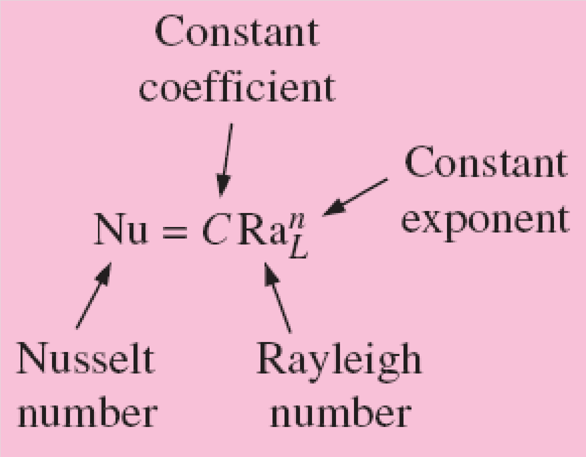
Rayleigh number

The constants C and n depend on the *geometry* of the surface and the *flow regime*, which is characterized by the range of the Rayleigh number.

The value of n is usually 1/4 for laminar flow and 1/3 for turbulent flow.

All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T_\infty)/2$.

$$\dot{Q}_{\mathrm{conv}} = hA_s(T_s - T_\infty) \qquad \mathrm{(W)}$$



Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant n multiplied by another constant C, both of which are determined experimentally.

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

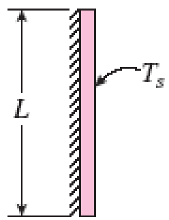
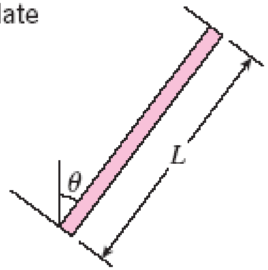
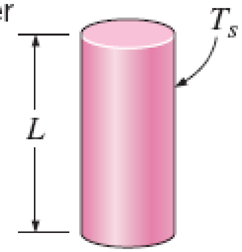
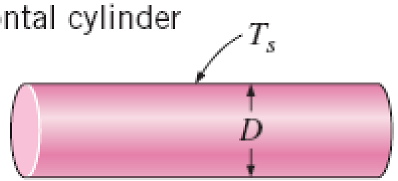
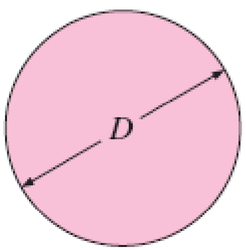
Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^{10}–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ **(9–19)** $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ **(9–20)** $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ **(9–21)** (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos\theta$ for $0 < \theta < 60°$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ **(9–22)** $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ **(9–23)** $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ **(9–24)**

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \dfrac{0.387Ra_D^{1/6}}{[1+(0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \dfrac{0.589Ra_D^{1/4}}{[1+(0.469/Pr)^{9/16}]^{4/9}}$

Vertical Plates (T_s = constant)

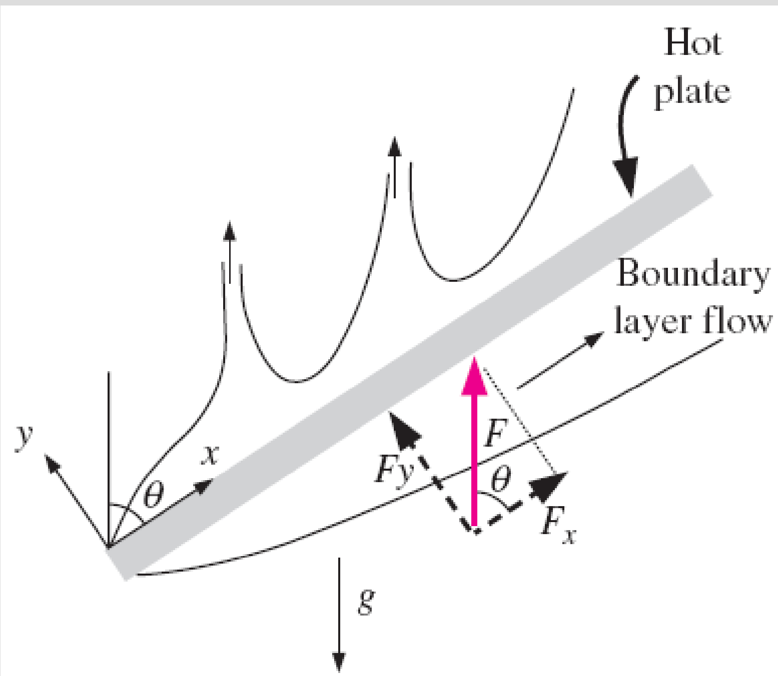
For a vertical flat plate, the characteristic length is the plate height L. In Table 9–1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one (Eq. 9–21) recommended by Churchill and Chu (1975) since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of $10^{-1} < Ra_L < 10^9$.

Vertical Plates (q_s = constant)

The relations for isothermal plates in the table can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number.

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \qquad \dot{Q} = \dot{q}_s A_s$$

Inclined Plates



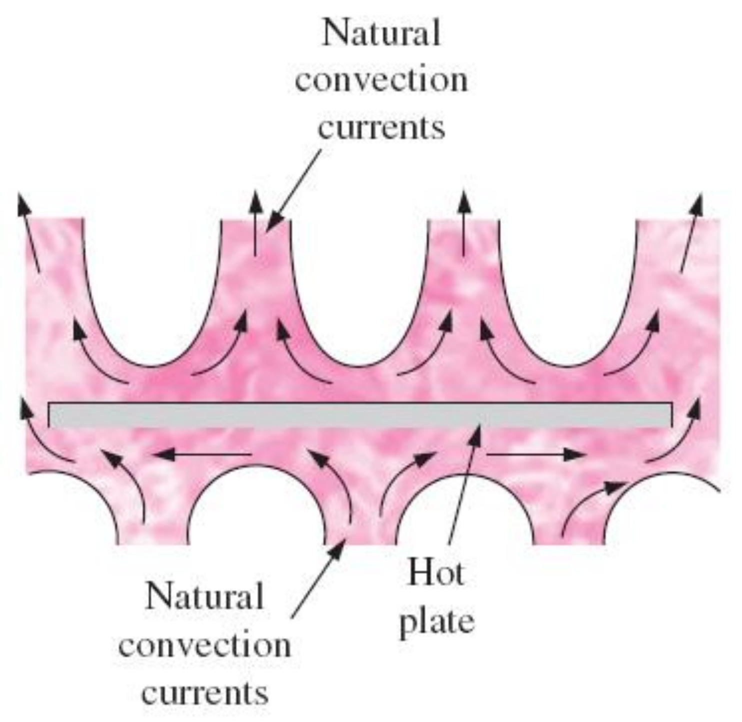
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

In a hot plate in a cooler environment for the lower surface of a hot plate, the convection currents are weaker, and the rate of heat transfer is lower relative to the vertical plate case.

On the upper surface of a hot plate, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs.

Horizontal Plates



Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise.

If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer.

But if the hot surface is facing downward, the plate blocks the heated fluid that tends to rise, impeding heat transfer.

The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

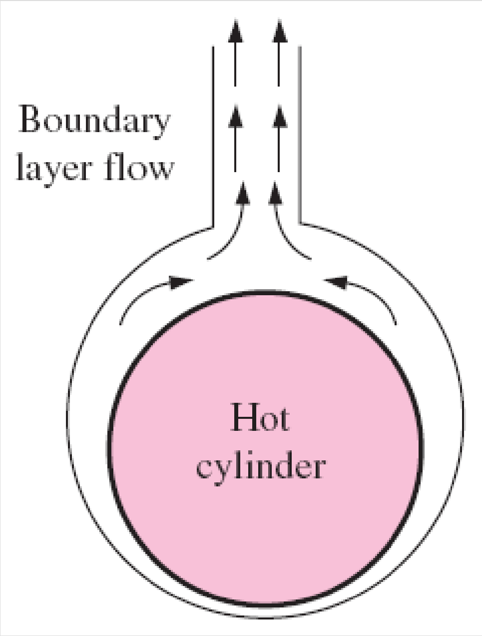
$$L_c = \frac{A_s}{p}$$

Characteristic length

$L_c = a/4$ for a horizontal square surface of length a

$L_c = D/4$ for a horizontal circular surface of diameter D

Horizontal Cylinders and Spheres



Natural convection flow over a horizontal hot cylinder.

The boundary layer over a hot horizontal cylinder starts to develop at the bottom, increasing in thickness along the circumference, and forming a rising plume at the top.

Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar.

The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

EXAMPLE 9–1 Heat Loss from Hot Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe shown in Figure 9–13 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

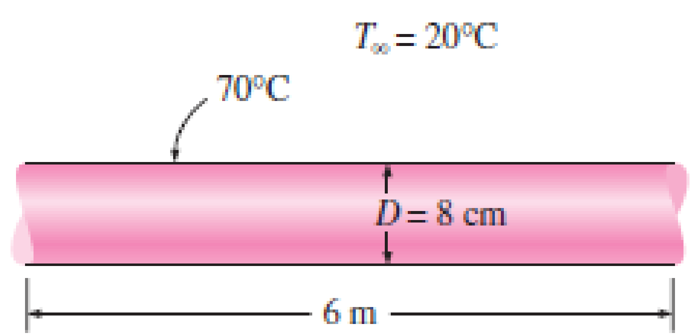


FIGURE 9–13
Schematic for Example 9–1.

SOLUTION A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–15)

$$k = 0.02699 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7241$$

$$\nu = 1.749 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{318 \text{ K}}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08$ m. Then the Rayleigh number becomes

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.749 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7241) = 1.869 \times 10^6$$

The natural convection Nusselt number in this case can be determined from Eq. 9-25 to be

$$\text{Nu} = \left\{0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1869 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}}\right\}^2$$

$$= 17.40$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.02699 \text{ W/m} \cdot °\text{C}}{0.08 \text{ m}}(17.40) = 5.869 \text{ W/m} \cdot °\text{C}$$

$$A_s = \pi DL = \pi(0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.869 \text{ W/m}^2 \cdot °\text{C})(1.508 \text{ m}^2)(70 - 20)°\text{C} = 443 \text{ W}$$

Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.

EXAMPLE 9–2 **Cooling of a Plate in Different Orientations**

Consider a 0.6-m × 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Figure 9–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.

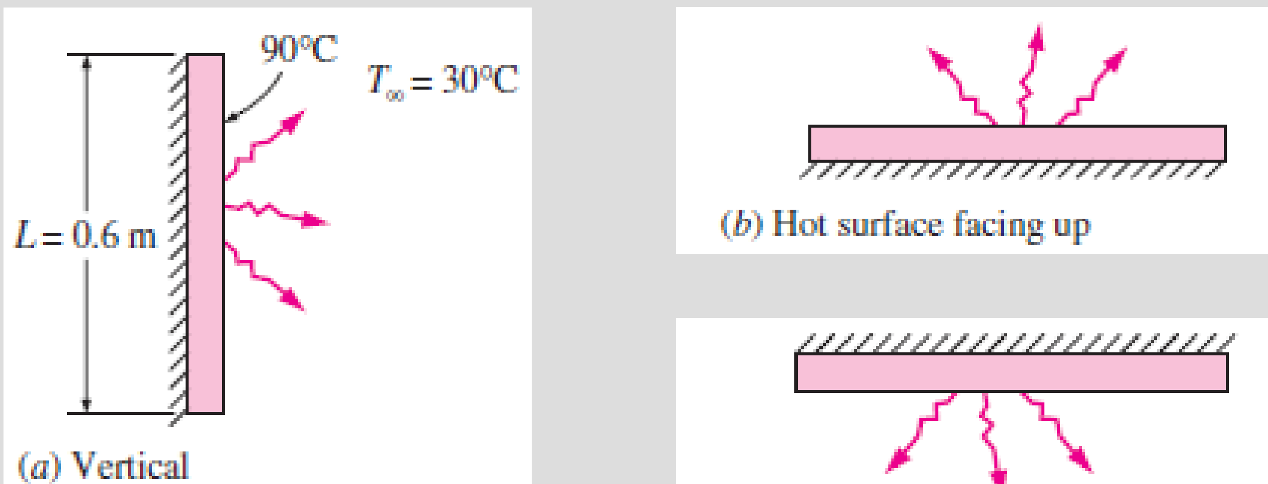


FIGURE 9–15
Schematic for Example 9–2.

SOLUTION A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7202$$

$$\nu = 1.896 \times 10^{-5} \text{ m}^2\text{/s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$

Analysis (*a*) *Vertical*. The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.722) = 7.656 \times 10^8$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\text{Nu} = \left\{0.825 + \frac{0.387 \text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4$$

Note that the simpler relation Eq. 9-19 would give Nu = 0.59 $Ra_L^{1/4}$ = 98.14, which is 13 percent lower. Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.02808 \text{ W/m}\cdot{}^\circ\text{C}}{0.6 \text{ m}}(113.4) = 5.306 \text{ W/m}^2\cdot{}^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306 \text{ W/m}^2\cdot{}^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{115\ W}$$

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.15 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$\text{Nu} = 0.54\ \text{Ra}_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.0280 \text{ W/m}\cdot{}^\circ\text{C}}{0.15 \text{ m}}(31.76) = 5.946 \text{ W/m}^2\cdot{}^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946 \text{ W/m}^2\cdot{}^\circ\text{C})(0.36 \text{ m}^2)(90 - 30)^\circ\text{C} = \mathbf{128\ W}$$

(*c*) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\text{Nu} = 0.27\,\text{Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02808\ \text{W/m}\cdot{}^\circ\text{C}}{0.15\ \text{m}}(15.86) = 2.973\ \text{W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973\ \text{W/m}^2\cdot{}^\circ\text{C})(0.36\ \text{m}^2)(90 - 30)^\circ\text{C} = \mathbf{64.2\ W}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is "trapped" under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

Natural Convection Cooling of Finned Surfaces (T_s = constant)

Finned surfaces of various shapes, called *heat sinks*, are frequently used in the cooling of electronic devices.

Energy dissipated by these devices is transferred to the heat sinks by conduction and from the heat sinks to the ambient air by natural or forced convection, depending on the power dissipation requirements.

Natural convection is the preferred mode of heat transfer since it involves no moving parts, like the electronic components themselves.

$$\text{Ra}_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2}\text{Pr}$$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \text{Ra}_S\frac{L^3}{S^3}$$

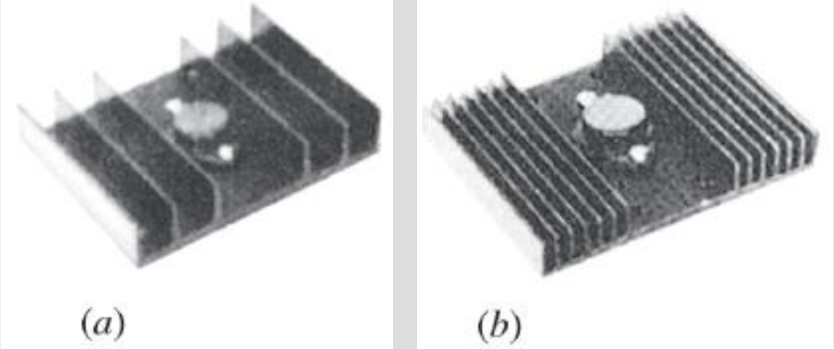
Characteristic lengths

S fin spacing

L fin height

$$T_s = \text{constant:} \qquad \text{Nu} = \frac{hS}{k} = \left[\frac{576}{(\text{Ra}_S S/L)^2} + \frac{2.873}{(\text{Ra}_S S/L)^{0.5}}\right]^{-0.5}$$

for vertical isothermal parallel plates



Heat sinks with (*a*) widely spaced and (*b*) closely packed fins.

Widely spaced: Smaller surface area but higher heat transfer coefficient

Closely packed: Higher surface area but smaller heat transfer coefficient

There must be an *optimum spacing* that maximizes the natural convection heat transfer from the heat sink.

When the fins are essentially isothermal and the fin thickness *t* is small relative to the fin spacing *S*, the optimum fin spacing for a vertical heat sink is

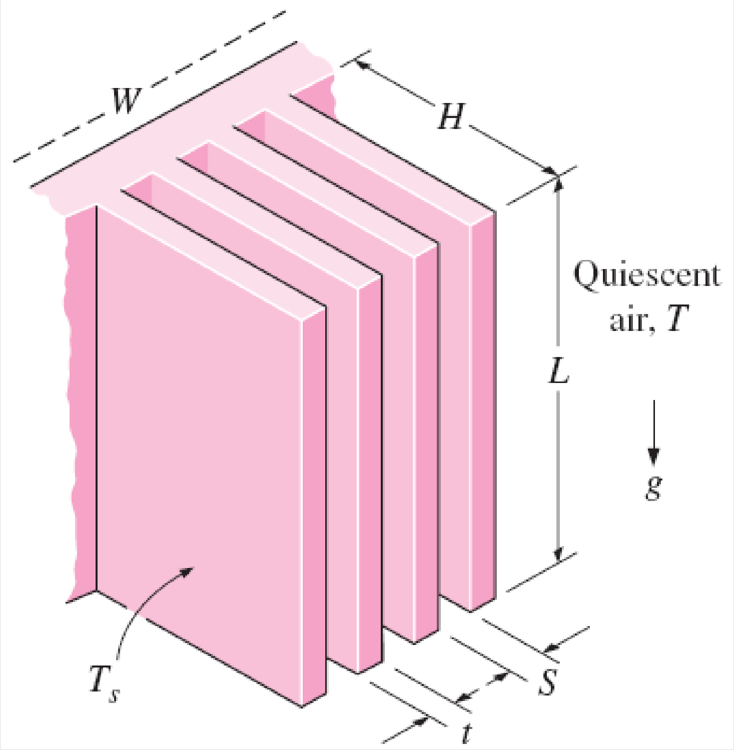
T_s = constant:
$$S_{opt} = 2.714\left(\frac{S^3 L}{Ra_S}\right)^{0.25} = 2.714\,\frac{L}{Ra_L^{0.25}}$$

$S = S_{opt}$:
$$Nu = \frac{hS_{opt}}{k} = 1.307$$

$$n = W/(S + t) \approx W/S$$

$$\dot{Q} = h(2nLH)(T_s - T_\infty)$$

All fluid properties are to be evaluated at the average temperature $T_{avg} = (T_s + T_\infty)/2$.



Various dimensions of a finned surface oriented vertically.

Natural Convection Cooling of Vertical PCBs (q_s = constant)

Arrays of printed circuit boards used in electronic systems can often be modeled as parallel plates subjected to uniform heat flux. The plate temperature in this case increases with height, reaching a maximum at the upper edge of the board.

$$\mathrm{Ra}_S^* = \frac{g\beta \dot{q}_s S^4}{k\nu^2}\mathrm{Pr}$$

$$\mathrm{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\mathrm{Ra}_S^* S/L} + \frac{2.51}{(\mathrm{Ra}_L^* S/L)^{0.4}}\right]^{-0.5}$$

$\dot{q}_s$ = constant: $$S_{\mathrm{opt}} = 2.12\left(\frac{S^4 L}{\mathrm{Ra}_S^*}\right)^{0.2}$$

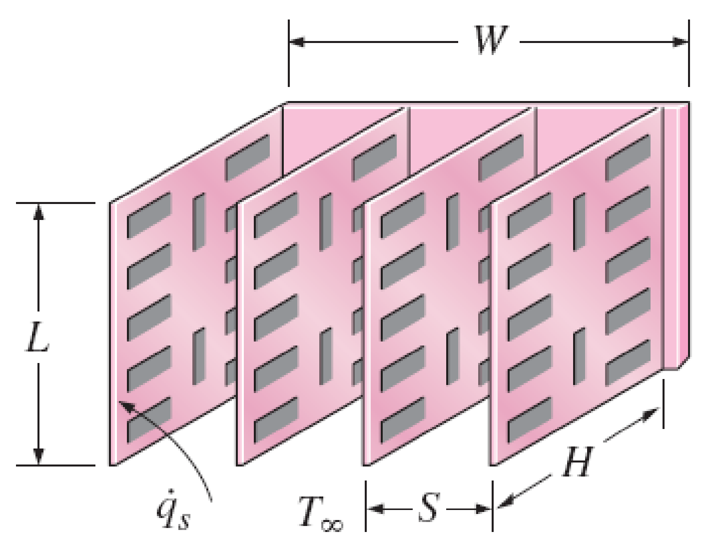
$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s(2nLH)$$

$n = W/(S+t) \approx W/S$ number of plates

The critical surface T_L that occurs at the upper edge of the plates is determined from

$$\dot{q}_s = h_L(T_L - T_\infty)$$

All fluid properties are to be evaluated at the average temperature $T_{avg} = (T_s + T_\infty)/2$.



Arrays of vertical printed circuit boards (PCBs) cooled by natural convection.

Mass Flow Rate through the Space between Plates

The magnitude of the natural convection heat transfer is directly related to the mass flow rate of the fluid, which is established by the dynamic balance of two opposing effects: *buoyancy* and *friction.*

The fins of a heat sink introduce both effects: *inducing extra buoyancy* as a result of the elevated temperature of the fin surfaces and *slowing down the fluid* by acting as an added obstacle on the flow path.

As a result, increasing the number of fins on a heat sink can either enhance or reduce natural convection, depending on which effect is dominant.

The buoyancy-driven fluid flow rate is established at the point where these two effects balance each other.

The friction force increases as more and more solid surfaces are introduced, seriously disrupting fluid flow and heat transfer. Heat sinks with closely spaced fins are not suitable for natural convection cooling.

When the heat sink involves widely spaced fins, the shroud does not introduce a significant increase in resistance to flow, and the buoyancy effects dominate. As a result, heat transfer by natural convection may improve, and at a fixed power level the heat sink may run at a lower temperature.

EXAMPLE 9–3 Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile (Fig. 9–20). The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.

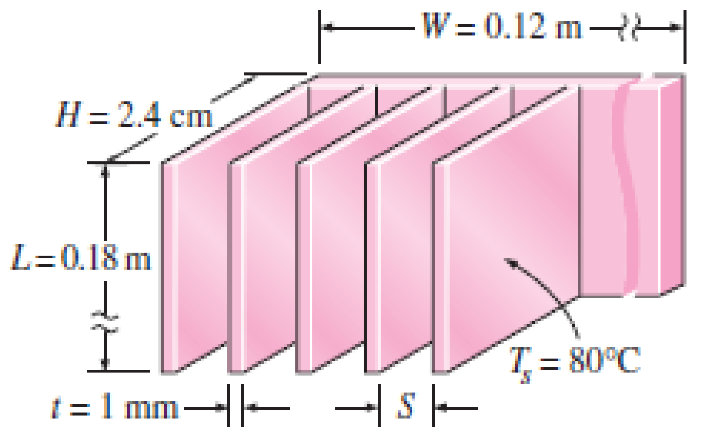


FIGURE 9–20
Schematic for Example 9–3.

SOLUTION A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The atmospheric pressure at that location is 1 atm. **4** The thickness t of the fins is very small relative to the fin spacing S so that Eq. 9-32 for optimum fin spacing is applicable. **5** All fin surfaces are isothermal at base temperature.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (80 + 30)/2 = 55°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02772 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7215$$
$$\nu = 1.846 \times 10^{-5} \text{ m}^2\text{/s} \qquad \beta = 1/T_f = 1/328 \text{ K}$$

Analysis We take the characteristic length to be the length of the fins in the vertical direction (since we do not know the fin spacing). Then the Rayleigh number becomes

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}$$
$$= \frac{(981 \text{ m/s}^2)[1/(328 \text{ K})](80 - 30 \text{ K})(0.18 \text{ m})^3}{(1.846 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.7215) = 1.846 \times 10^7$$

The optimum fin spacing is determined from Eq. 7-32 to be

$$S_{\text{opt}} = 2.714\frac{L}{\text{Ra}_L^{0.25}} = 2.714\frac{0.8 \text{ m}}{(1.846 \times 10^7)^{0.25}} = 7.45 \times 10^{-3} \text{ m} = \mathbf{7.45 \text{ mm}}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are

$$n = \frac{W}{S + t} = \frac{0.12\ \text{m}}{(0.00745 + 0.0001)\ \text{m}} \approx 15\ \text{fins}$$

The convection coefficient for this optimum in spacing case is, from Eq. 9-33,

$$h = \text{Nu}_{\text{opt}} \frac{k}{S_{\text{opt}}} = 1.307 \frac{0.02772\ \text{W/m} \cdot \text{°C}}{0.00745\ \text{m}} = 0.2012\ \text{W/m}^2 \cdot \text{°C}$$

Then the rate of natural convection heat transfer becomes

$$\begin{aligned}\dot{Q} &= hA_s(T_s - T_\infty) = h(2nLH)(T_s - T_\infty)\\ &= (0.2012\ \text{W/m}^2 \cdot \text{°C})[2 \times 15(0.18\ \text{m})(0.024\ \text{m})](80 - 30)\text{°C} = \mathbf{1.30\ W}\end{aligned}$$

Therefore, this heat sink can dissipate heat by natural convection at a rate of 1.30 W.

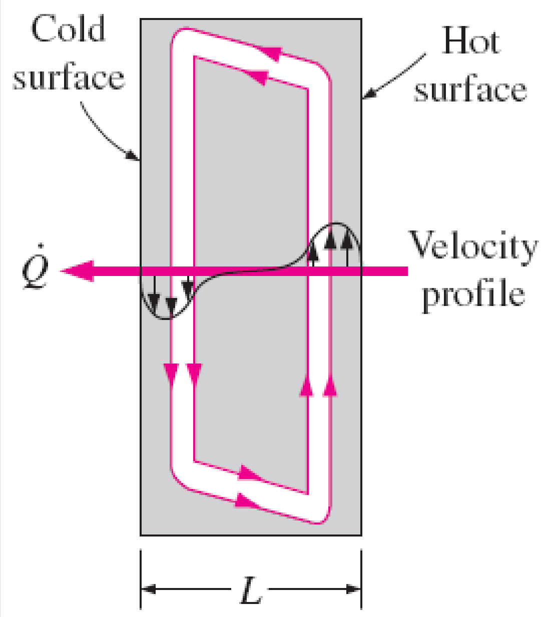
NATURAL CONVECTION INSIDE ENCLOSURES

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure.

$$\mathrm{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\mathrm{Pr}$$

L_c charecteristic length: the distance between the hot and cold surfaces

T_1 and T_2: the temperatures of the hot and cold surfaces



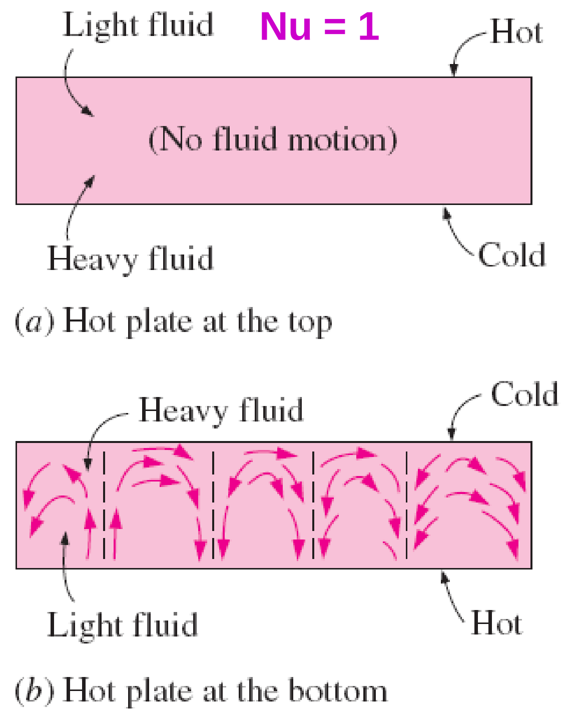
Convective currents in a vertical rectangular enclosure.

Ra > 1708, natural convection currents

Ra > 3×10^5, turbulent fluid motion

Fluid properties at

$$T_{avg} = (T_1 + T_2)/2.$$



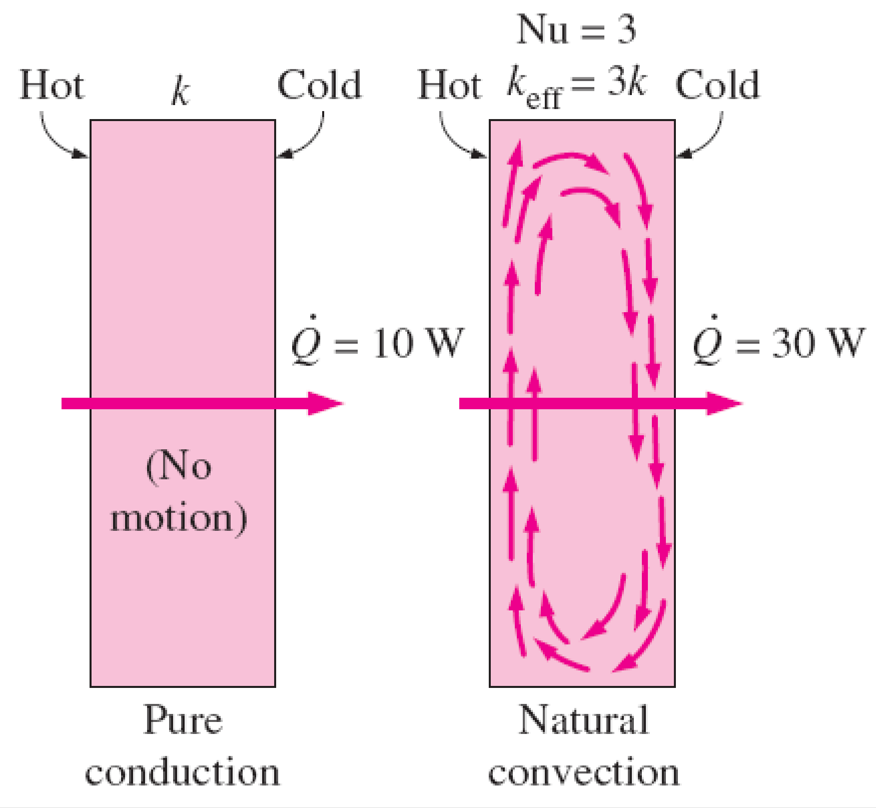
Convective currents in a horizontal enclosure with (*a*) hot plate at the top and (*b*) hot plate at the bottom.

Effective Thermal Conductivity

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s\frac{T_1 - T_2}{L_c}$$

$$h = k\text{Nu}/L_c$$

$$\dot{Q}_{\text{cond}} = kA_s\frac{T_1 - T_2}{L_c}$$



$$k_{\text{eff}} = k\text{Nu}$$

effective thermal conductivity

The fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents.

Nu = 1, the effective thermal conductivity of the enclosure is equal to the conductivity of the fluid. This case corresponds to pure conduction.

Numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of $\text{Nu} = C\text{Ra}^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios.

A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction.*

Horizontal Rectangular Enclosures

$$Nu = 0.195 Ra_L^{1/4} \qquad 10^4 < Ra_L < 4 \times 10^5$$

$$Nu = 0.068 Ra_L^{1/3} \qquad 4 \times 10^5 < Ra_L < 10^7$$

For horizontal enclosures that contain air, These relations can also be used for other gases with $0.5 < Pr < 2$.

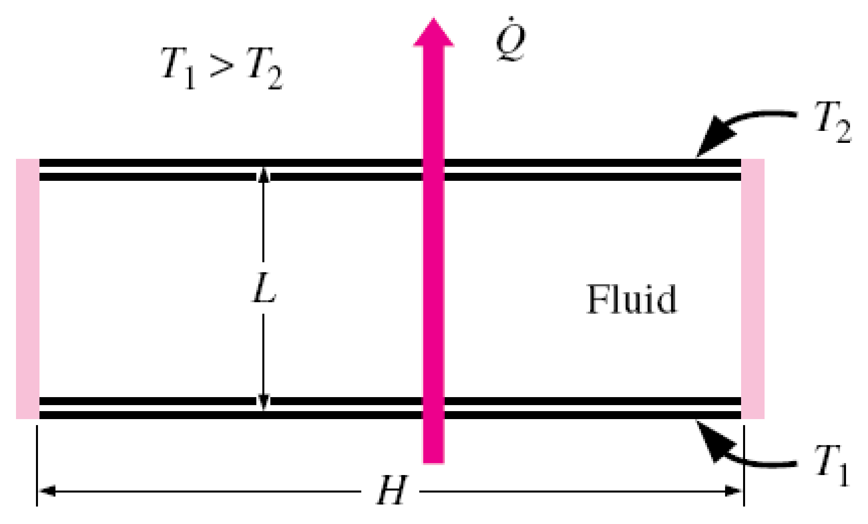
$$Nu = 0.069 Ra_L^{1/3} Pr^{0.074} \qquad 3 \times 10^5 < Ra_L < 7 \times 10^9$$

For water, silicone oil, and mercury

$$Nu = 1 + 1.44 \left[1 - \frac{1708}{Ra_L}\right]^+ + \left[\frac{Ra_L^{1/3}}{18} - 1\right]^+ \qquad Ra_L < 10^8$$

$[\ ]^+$ only positive values to be used

Based on experiments with air. It may be used for liquids with moderate Prandtl numbers for $Ra_L < 10^5$.



When the hotter plate is at the top, $Nu = 1$.

A horizontal rectangular enclosure with isothermal surfaces.

Inclined Rectangular Enclosures

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L \cos\theta}\right]^{+}\left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}_L \cos\theta}\right) + \left[\frac{(\text{Ra}_L \cos\theta)^{1/3}}{18} - 1\right]^{+}$$

$\text{Ra}_L < 10^5,\ 0 < \theta < 70°,\ \text{and}\ H/L \geq 12.$

$$\text{Nu} = \text{Nu}_{\theta=0°}\left(\frac{\text{Nu}_{\theta=90°}}{\text{Nu}_{\theta=0°}}\right)^{\theta/\theta_{cr}} (\sin\theta_{cr})^{\theta/(4\theta_{cr})} \qquad 0° < \theta < \theta_{cr}$$

$H/L < 12$

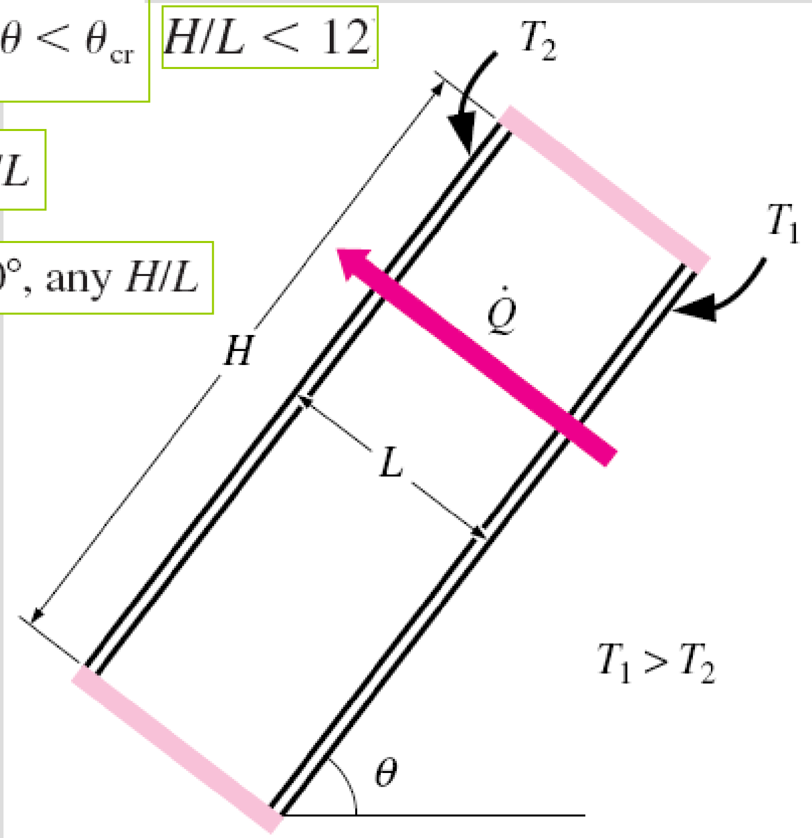
$$\text{Nu} = \text{Nu}_{\theta=90°}(\sin\theta)^{1/4} \qquad \theta_{cr} < \theta < 90°,\ \text{any } H/L$$

$$\text{Nu} = 1 + (\text{Nu}_{\theta=90°} - 1)\sin\theta \qquad 90° < \theta < 180°,\ \text{any } H/L$$

TABLE 9–2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
>12	70°



An inclined rectangular enclosure with isothermal surfaces.

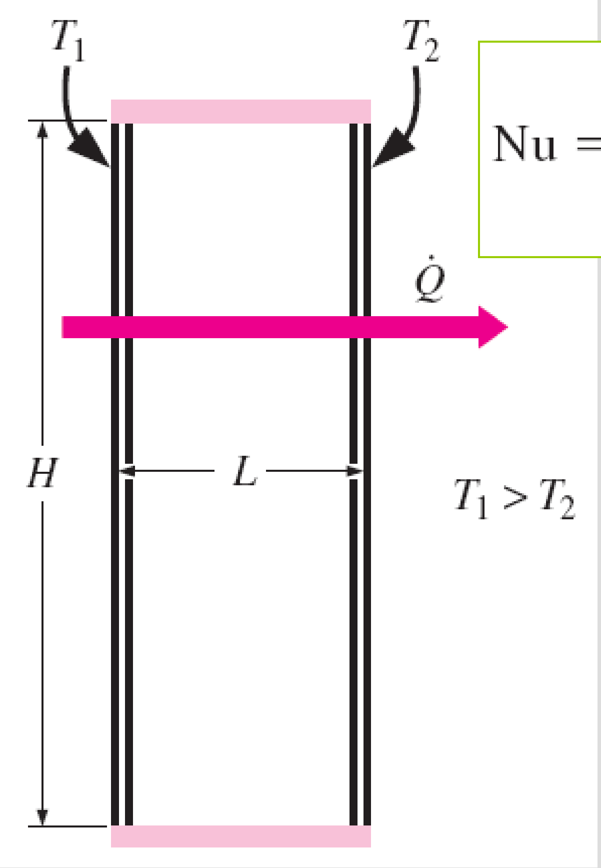
Vertical Rectangular Enclosures

$$\text{Nu} = 0.18\left(\frac{\text{Pr}}{0.2+\text{Pr}}\text{Ra}_L\right)^{0.29}$$

$1 < H/L < 2$
any Prandtl number
$\text{Ra}_L\,\text{Pr}/(0.2+\text{Pr}) > 10^3$

$$\text{Nu} = 0.22\left(\frac{\text{Pr}}{0.2+\text{Pr}}\text{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$$

$2 < H/L < 10$
any Prandtl number
$\text{Ra}_L < 10^{10}$



$$\text{Nu} = 0.42\text{Ra}_L^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$10 < H/L < 40$
$1 < \text{Pr} < 2\times10^4$
$10^4 < \text{Ra}_L < 10^7$

$$\text{Nu} = 0.46\text{Ra}_L^{1/3}$$

$1 < H/L < 40$
$1 < \text{Pr} < 20$
$10^6 < \text{Ra}_L < 10^9$

Again, all fluid properties are to be evaluated at the average temperature $(T_1+T_2)/2$.

A vertical rectangular enclosure with isothermal surfaces.

Concentric Cylinders

The rate of heat transfer through the annular space between the cylinders by natural convection per unit length

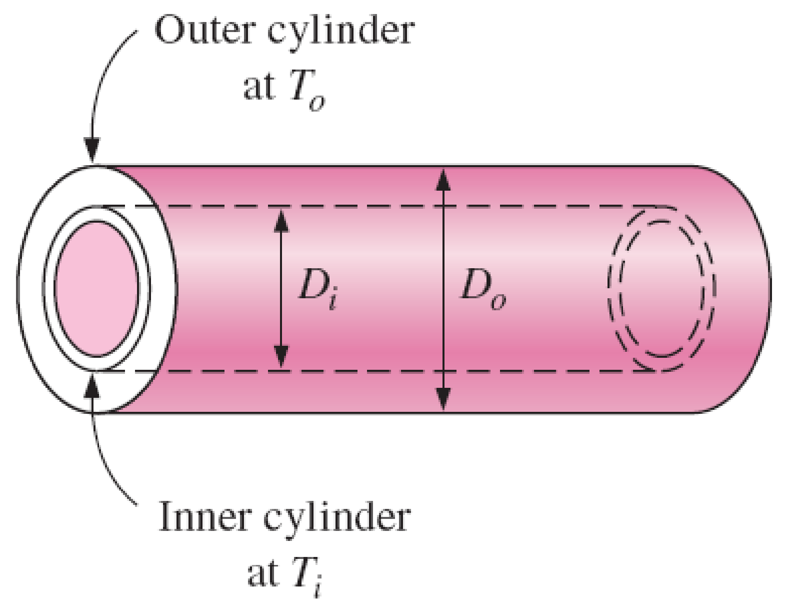
$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) \qquad (\text{W/m})$$

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4}$$

Characteristic length

$$L_c = (D_o - D_i)/2$$

$$0.70 \le \text{Pr} \le 6000 \text{ and } 10^2 \le F_{\text{cyl}}\text{Ra}_L \le 10^7$$



Two concentric horizontal isothermal cylinders.

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

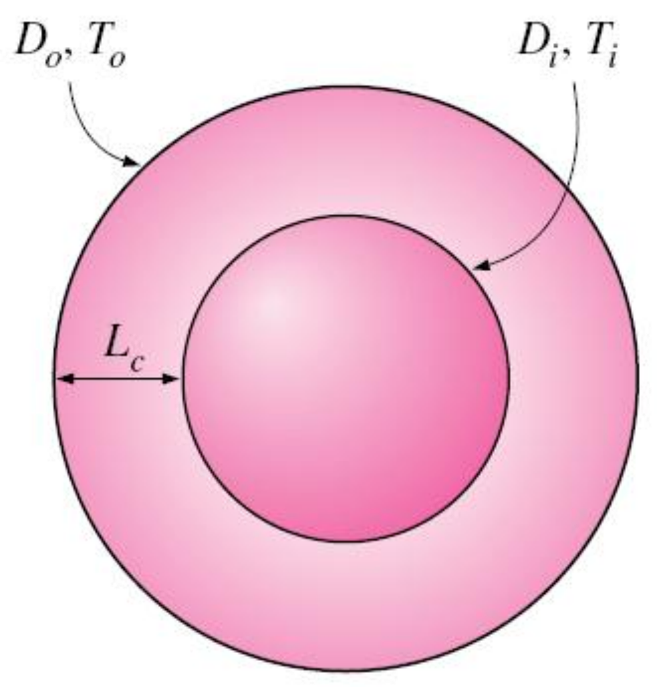
the geometric factor for concentric cylinders

For $F_{\text{cyl}}\text{Ra}_L < 100$, natural convection currents are negligible and thus $k_{\text{eff}} = k$.

Note that k_{eff} cannot be less than k, and thus we should set $k_{\text{eff}} = k$ if $k_{\text{eff}}/k < 1$.

The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

Concentric Spheres



Two concentric isothermal spheres.

Characteristic length

$$L_c = (D_o - D_i)/2$$

$$\dot{Q} = k_{\text{eff}} \frac{\pi D_i D_o}{L_c}(T_i - T_o) \qquad \text{(W)}$$

$$\frac{k_{\text{eff}}}{k} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}$$

$$0.70 \le \text{Pr} \le 4200 \text{ and } 10^2 \le F_{\text{sph}}\text{Ra}_L \le 10^4$$

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5}$$

If $k_{eff}/k < 1$, we should set $k_{eff} = k$.

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction) and radiation. Radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities.

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_{surr} is

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \qquad \text{(W)}$$

$\sigma = 5.67 \times 10^8\ \text{W/m}^2\cdot\text{K}^4$
Stefan–Boltzmann constant

Radiation heat transfer between two large parallel plates is

$$\dot{Q}_{\text{rad}} = \frac{\sigma A_s(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{\text{effective}}\,\sigma A_s(T_1^4 - T_2^4) \qquad \text{(W)}$$

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

When $T_\infty < T_s$ and $T_{\text{surr}} > T_s$, convection and radiation heat transfers are in opposite directions and subtracted from each other.

EXAMPLE 9–4 **Heat Loss through a Double-Pane Window**

The vertical 0.8-m-high, 2-m-wide double-pane window shown in Fig. 9–29 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

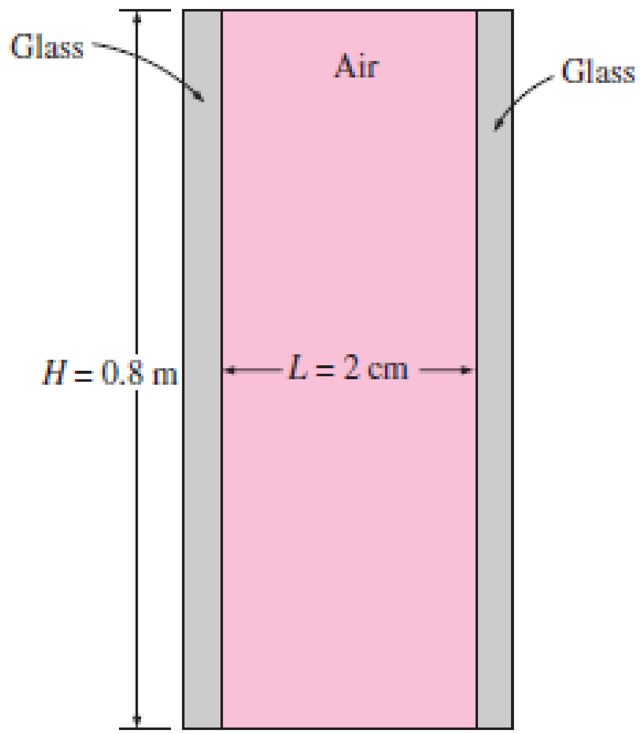


FIGURE 9–29
Schematic for Example 9–4.

SOLUTION Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{ave} = (T_1 + T_2)/2 = (12 + 2)/2 = 7°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02416 \text{ W/m} \cdot {}^\circ\text{C} \qquad \text{Pr} = 0.7344$$

$$\nu = 1.399 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{280 \text{ K}}$$

Analysis We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses, $L = 0.02$ m. Then the Rayleigh number becomes

$$\text{Ra}_L = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(280 \text{ K})](12 - 2 \text{ K})(0.02 \text{ m})^3}{(1.399 \times 10^{-5} \text{m}^2/\text{s})^2}(0.7344) = 1.051 \times 10^4$$

The aspect ratio of the geometry is $H/L = 0.8/0.02 = 40$. Then the Nusselt number in this case can be determined from Eq. 9-54 to be

$$\text{Nu} = 0.42\text{Ra}_L^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$$= 0.42(1.051 \times 10^4)^{1/4}(0.7344)^{0.012}\left(\frac{0.8}{0.02}\right)^{-0.3} = 1.401$$

Then,

$$A_s = H \times W = (0.8 \text{ m})(2 \text{ m}) = 1.6 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s \frac{T_1 - T_2}{L}$$

$$= (0.02416 \text{ W/m} \cdot {}^\circ\text{C})(1.401)(1.6 \text{ m}^2) \frac{(12 - 2)^\circ\text{C}}{0.02 \text{ m}} = \mathbf{27.1 \ W}$$

Therefore, heat will be lost through the window at a rate of 27.1 W.

EXAMPLE 9–5 **Heat Transfer through a Spherical Enclosure**

The two concentric spheres of diameters $D_i = 20$ cm and $D_o = 30$ cm shown in Fig. 9–30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320$ K and $T_o = 280$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

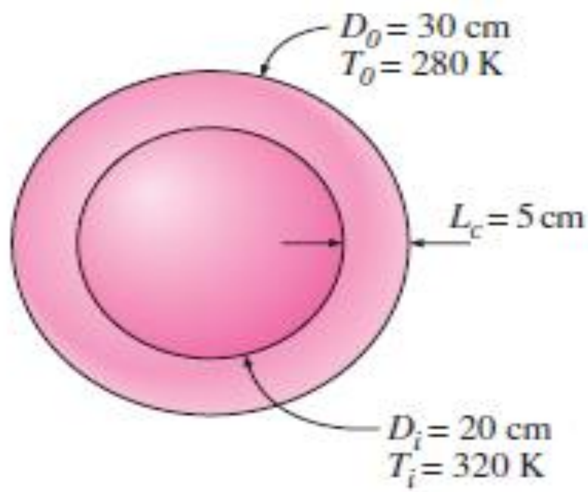


FIGURE 9–30
Schematic for Example 9–5.

SOLUTION Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{ave} = (T_i + T_o)/2 = (320 + 280)/2 = 300\text{ K} = 27°\text{C}$ and 1 atm pressure are (Table A-15)

$$k = 0.02566\text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7290$$

$$\nu = 1.580 \times 10^{-5}\text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{300\text{ K}}$$

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05\text{ m}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\text{ m/s}^2)[1/(300\text{ K})](320 - 280\text{ K})(0.05\text{ m})^3}{(1.58 \times 10^{-5}\text{ m}^2/\text{s})^2}(0.729) = 4.776 \times 10^5$$

The effective thermal conductivity is

$$F_{sph} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}$$
$$= \frac{0.05\text{ m}}{[(0.2\text{ m})(0.3\text{ m})]^4[(0.2\text{ m}^{-7/5} + (0.3\text{ m})^{-7/5}]^5} = 0.005229$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}$$

$$= 0.74(0.02566\ \text{W/m}\cdot{}^\circ\text{C})\left(\frac{0.729}{0.861 + 0.729}\right)(0.005229 \times 4.776 \times 10^5)^{1/4}$$

$$= 0.1104\ \text{W/m}\cdot{}^\circ\text{C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o)$$

$$= (0.1104\ \text{W/m}\cdot{}^\circ\text{C})\pi\left(\frac{(0.2\ \text{m})(0.3\ \text{m})}{0.05\ \text{m}}\right)(320 - 280)\text{K} = \mathbf{16.7\ W}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

EXAMPLE 9–6 Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in. enclosed in a concentric thin glass tube of 4-in.-diameter (Fig. 9–31). Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

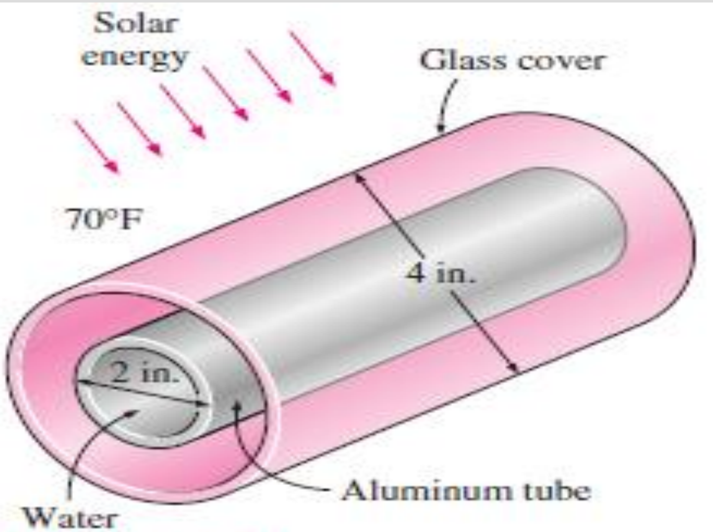


FIGURE 9–31
Schematic for Example 9–6.

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** Heat loss by radiation is negligible.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of (70 + 110)/2 = 90°F (Table A-15E),

$$k = 0.01505 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F} \qquad \text{Pr} = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2/\text{h} = 1.753 \times 10^{-4} \text{ ft}^2/\text{s} \qquad \beta = \frac{1}{T_{\text{ave}}} = \frac{1}{550 \text{ K}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12 \text{ ft})(1 \text{ ft}) = 1.047 \text{ ft}^2 \qquad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 100°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\mathrm{Ra}_{D_o} = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\mathrm{Pr}$$

$$= \frac{(32.2\ \mathrm{ft/s^2})[1/(550\ \mathrm{R})](110 - 70\ \mathrm{R})(4/12\ \mathrm{ft})^3}{(1.753 \times 10^{-4}\ \mathrm{ft^2/s})^2}(0.7275) = 2.054 \times 10^6$$

$$\mathrm{Nu} = \left\{0.6 + \frac{0.387\ \mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.054 \times 10^6)^{1/6}}{[1 + (0.559/0.7275)^{9/16}]^{8/27}}\right\}^2$$

$$= 17.89$$

$$h_o = \frac{k}{D_0}\mathrm{Nu} = \frac{0.0150\ \mathrm{Btu/h \cdot ft \cdot {}^\circ F}}{4/12\ \mathrm{ft}}(17.89) = 0.8075\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F}$$

$$\dot{Q}_o = h_o A_o (T_o - T_\infty) = (0.8075\ \mathrm{Btu/h \cdot ft^2 \cdot {}^\circ F})(1.047\ \mathrm{ft^2})(110 - 70)^\circ\mathrm{F}$$

$$= 33.8\ \mathrm{Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures, the glass cover temperature corresponding to 30 Btu/h is determined to be 106°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1 \text{ in.} = 1/12 \text{ ft}$$

We start the calculations by assuming the tube temperature to be 200°F, and thus an average temperature of (106 + 200)/2 = 154°F = 614 R. This gives

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}$$

$$= \frac{(32.2 \text{ ft/s}^2)[1/614 \text{ R}](200 - 106 \text{ R})(1/12 \text{ ft})^3}{(2.117 \times 10^{-4} \text{ ft}^2\text{/s})^2}(0.7184) = 4.579 \times 10^4$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

$$= \frac{[\ln(4/2)]^4}{(1/12 \text{ ft})^3[(2/12 \text{ ft})^{-3/5} + (4/12 \text{ ft})^{-3/5}]^5} = 0.1466$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4}$$

$$= 0.386(0.01653 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})\left(\frac{0.7184}{0.861 + 0.7184}\right)(0.1466 \times 4.579 \times 10^4)^{1/4}$$

$$= 0.04743 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$

$$= \frac{2\pi(0.04743 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})}{\ln(4/2)}(200 - 106)^\circ\text{F} = 40.4 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 200°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **180°F.** Therefore, the tube will reach an equilibrium temperature of 180°F when the pump fails.